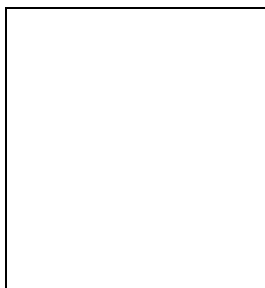


## LEPTON MIXING: SMALL, LARGE, MAXIMAL?

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The SuperKamiokande data on atmospheric neutrinos imply that  $\nu_\mu$  has large (or even maximal) mixing. It is still open question whether this mixing is the flavor one or mixing with singlet state (sterile neutrino). Several tests exist to establish the channel of atmospheric neutrino oscillations. Large mixing can be a property of the second and third generations, so that the scheme with single large mixing is realized. It can be a generic property of all leptons, thus leading to the bi-large or threefold large mixing schemes. The ambiguity will be resolved by identification of solution of the solar neutrino problem. In this review we consider phenomenology of various mixing schemes. The key measurements which will allow us to make significant step in reconstruction of the neutrino mass and flavor spectrum are discussed.

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# 1 Introduction

During 736 days of observations the SuperKamiokande (SK) collaboration has detected more than 6300 atmospheric neutrinos<sup>1</sup>. Other detectors have recorded about 2000 events<sup>2,3,4,5,6</sup>. With this results we are entering new stage of high statistics and high precision studies. Events detected so far differ by topology ( $\alpha$ ): one (Cherenkov) ring  $e$ -like and  $\mu$ -like events, two ring events (where the so called  $\pi^0$  events are of special interest), multi-ring events, upward-going muons etc.. The energy,  $E$ , and the zenith angle,  $\Theta$ , dependences of events of each topology have been measured. That is, numbers of events,  $N_\alpha(E, \Theta)$ , of the type  $\alpha$  have been found in various energy and zenith angle bins.

With increase of statistics various data from the SuperKamiokande as well as from SOUDAN<sup>4</sup> and MACRO<sup>5</sup> show better consistency. Their implications converge to the same oscillation interpretation. In particular, the SOUDAN observes the up - down asymmetry of the tracks (muons). The best fit of the MACRO data on through-going muons corresponds to oscillation interpretation; comparison of stopping to through-going muons further confirms the oscillations. There is also better internal consistency of the SK data themselves. One change is rather significant: the rate of the  $e$ -like events has decreased in the last series of observations, and correspondingly, the double ratio:  $(R_{\mu/e})_{exp}/(R_{\mu/e})_{MC}$  of the experimental and the simulated  $\nu_\mu$  to  $\nu_e$  ratios ( $R_{\mu/e}$ ) has increased. Further independent confirmation of the oscillations has been obtained from studies of the sub-dominant samples of events (two rings events, stopping/through-going etc.). The observation of the East - West (geomagnetic) effect<sup>7</sup> confirms overall picture of generation of the atmospheric neutrinos.

On the basis of this information we can say with high confidence level that

- The atmospheric *muon neutrinos oscillate*:  $\nu_\mu \leftrightarrow \nu_x$ , where  $\nu_x$  is in general a combination of the muon, electron, and probably, sterile neutrinos.
- The *oscillations are induced by neutrino mass* difference and vacuum mixing. Other interpretations<sup>8,9</sup> are disfavored<sup>10,11</sup>.
- The electron neutrino  $\nu_e$  is not a dominant component of  $\nu_x$ . Data favor  $\nu_\mu \leftrightarrow \nu_\tau$  as the main (dominating) mode of oscillation, although  $\nu_\mu \leftrightarrow \nu_s$ , as the dominant channel, is not excluded.

With high statistics we are now in position to make next step: to study sub-leading effects. We can approach the following three problems:

1. Identification of the channel of oscillations.
2. Clarification of the role of the  $\nu_e$  in oscillations of the atmospheric neutrinos. In particular, determination of the admixture of the electron neutrino in the heavy mass eigenstate,  $U_{e3}$ , can be done by studies of the multi-GeV  $e$ -like events.
3. Searches of oscillation effects driven by smallest  $\Delta m^2$  splitting responsible, presumably, for the solution of the solar neutrino problem. This can be done by precision studies of the sub-GeV  $e$ -like events.

The above effects are typically smaller than 10% of the main oscillation effect and samples of events sensitive to these effects are an order of magnitude smaller than total number of events.

Let us outline situation with solar neutrinos. After 708 days of observations the SK has detected more than 11500 events<sup>12</sup>. This opens a possibility to search for *signatures* of various oscillation solutions in the ranges of oscillation parameters determined by the fit of total rates. That is, the total event rates in all existing experiments. (This means that we have indeed started crucial checks of proposed solutions.) Among these signatures are:

- (i) Distortion of the recoil electron spectrum  $R(E) \equiv N(E)^{obs}/N(E)^{SSM}$ . Some distortion (deviation of  $R(E)$  from a constant) has already been observed. The main feature of the data is that below electron energy 13.5 MeV they are in agreement with undistorted spectrum. Above 13.5 MeV there

is about  $3\sigma$  excess whose interpretation is still unclear: It can be just statistical fluctuation, or it can be due to large flux of the *hep*-neutrinos, or due to effect of the vacuum oscillations with relatively large  $\Delta m^2$ . It will be important to see the result on the spectrum in the lowest bin 5.0 - 5.5 MeV.

(ii) The night-day asymmetry

$$A_{N/D} \equiv 2 \frac{N - D}{N + D} \approx \frac{N}{D} - 1, \quad (1)$$

where  $N$  ( $D$ ) is the nighttime (daytime) signal integrated over energies above 6.5 MeV and averaged over the year. The asymmetry is observed at the level 6% which differs from zero by  $1.6\sigma$ . Recent data from 825 days of observations<sup>13</sup> has increased this significance to  $2\sigma$ .

(iii) The zenith angle distribution of events averaged over the year:  $\bar{F}(\Theta)$ .

(iv) Seasonal variations of the signal. One can characterize it by the winter-summer asymmetry:

$$A_{W/S} \equiv 2 \frac{W - S}{W + S}, \quad (2)$$

where  $W$  and  $S$  are the signals averaged over winter time (November 15 - February 15) and summer time (May 15 - August 15) respectively. Important criteria can be obtained from the dependence of the asymmetry on the energy threshold. In fact, the SK sees strong (but statistically insignificant) enhancement of the variations with increase of the threshold. Being confirmed, it will allow to identify the solution.

The main conclusion from analysis of present situation is that *different datasets favor different solutions of the problem*. In particular, the N/D asymmetry and the zenith angle dependence indicate Large Mixing Angle (LMA) MSW solution. Seasonal variations and the spectrum distortion testify for the Vacuum Oscillation (VO) solution, whereas the total rates of signals prefer Small Mixing Angle (SMA) MSW solution. The significance of all these indications is about  $2\sigma$ , and clearly, more data is needed to make definite conclusion.

Concerning the third, LSND<sup>14</sup>, evidence of the neutrino mass and mixing, it seems that the oscillation interpretation of the LSND result can not be completely excluded by KARMEN<sup>15</sup>. Situation could be changed if the KARMEN will start to see some excess of events. However, in any case new experiments, like the MiniBoon<sup>16</sup>, are needed,

Other key results in the “game” are: (i) the bounds on neutrino mass and mixing which follow from reactor experiments CHOOZ<sup>17</sup> and Palo-Verde<sup>18</sup>, (ii) new bound on the effective Majorana mass from the neutrinoless double beta decay<sup>19</sup> and (iii) bound on neutrino masses inferred by the large scale structure of the Universe.

There are two related fundamental goals of these studies:

- Reconstruction of the neutrino mass spectrum and lepton mixing. Clearly, it is as fundamental as reconstruction of the quark mass spectrum and determination of the quark mixings.

Eventually, it will be of great importance to compare quark and lepton spectra and mixings. This will give certain insight to

- unification of the quarks and leptons;
- quark - lepton symmetry;
- mechanism of neutrino mass generation;
- origin of the fermion masses;
- new symmetries and new mass (energy) scales in Nature.

- Searches for new neutrino states, singlets of the standard model group. Although these new light fermions can couple in some way (*e.g.* via non-renormalizable interactions) with other SM

particles, their presence can be revealed in the neutrino physics only due to unique properties of neutrinos.

There are two main motivations for introduction of the sterile neutrinos: the first one is to explain all neutrino anomalies including the LSND result in terms of neutrino oscillations. The second is to explain the large lepton mixing itself (see below).

Notice that even if sterile neutrinos do not play major role in explanation of the neutrino problems, they may produce some sub-leading effects. And searches for possible presence of sterile neutrinos is of great importance. Existence of such fermions can lead us far beyond the Standard Model. Practically all extensions of the Standard model predict existence of the singlets. Open questions are their parameters: masses and mixing. The presence of sterile neutrinos with parameters which can give observable effects is rather plausible. Indeed, even Planck scale effects can produce the mass terms which lead to effects can be seen in solar and supernove neutrino fluxes<sup>20</sup>. The mixing mass term of singlet fermion with neutrinos can be of the order<sup>21</sup>

$$\bar{m} = \frac{vM}{M_{Pl}} \sim 10^{-4} \left( \frac{M}{1\text{TeV}} \right) \text{eV} , \quad (3)$$

where  $v$  is the vev of the higgs doublet, the mass  $M$  can be the gravitino mass in of the SUGRA,  $m_{3/2}$ <sup>21</sup>, or the  $\mu$  parameter of the SUSY<sup>22,23</sup>, or the scale of breaking of an additional gauge  $U(1)$  factor<sup>21,24</sup> etc.. Clearly, the mass scale in the denominator smaller than of  $M_{Pl}$  (which would imply new physics below the Planck scale) will produce larger mixing mass. Of course, crucial question is the one about mass of the singlet. The mass of the order  $M^2/M_{Pl}$  can lead to observable effects.

## 2 Small

The only possibility to explain the atmospheric neutrino data and still to keep small flavor mixings (similar to quark mixings) is to introduce the sterile neutrino which mixes strongly with  $\nu_\mu$ , so that the atmospheric neutrinos undergo  $\nu_\mu \leftrightarrow \nu_s$  oscillations (fig. 1). In this case the solar neutrino problem should be solved by the small mixing angle MSW conversion. There are two versions (in the schemes

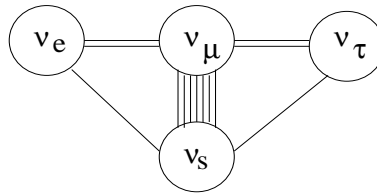


Figure 1: The small flavor mixing scheme. The number of lines which connect circles reflect a strength of mixing.

with only one sterile neutrino):

$$\nu_e \rightarrow \nu_\tau$$

which corresponds to the scheme<sup>25</sup> shown in fig. 2 , or

$$\nu_e \rightarrow \nu_\mu, \nu_s .$$

In the first case one can also explain the LSND result. The second case is characterized by mass hierarchy  $m_1 \ll m_2 \ll m_4 \ll m_3$ , where the heavy state  $\nu_3 \approx \nu_\tau$  can be in the eV- range thus contributing significantly to the HDM.  $\nu_\mu$  and  $\nu_s$  are strongly mixed in  $\nu_2$  and  $\nu_4$  whose masses are determined the solar and the atmospheric mass scales. (see<sup>32</sup> for details).

The key question is why sterile neutrino mixes strongly with  $\nu_\mu$  and its mixing with other neutrinos either absent or very small. The suppression of the  $\nu_e - \nu_s$  and  $\nu_\tau - \nu_s$  mixing can be related to hierarchy of the diagonal mass terms of the active neutrino components and closeness of the sterile neutrino mass to the mass of  $\nu_\mu$ .

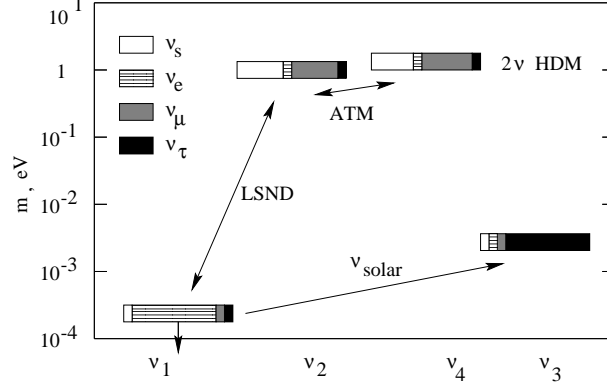


Figure 2: The neutrino masses and mixing scheme with small flavor mixing and one sterile neutrino. Boxes correspond to the mass eigenstates. The sizes of different regions in the boxes show admixtures of different flavors. Weakly hatched regions correspond to the electron flavor, strongly hatched regions depict the muon flavor, black regions present the tau flavor.

Let us consider generic signatures of the schemes which employ the  $\nu_\mu \leftrightarrow \nu_s$  solution of the atmospheric neutrino problem and small mixing MSW solution of the solar neutrino problem. The  $\nu_\mu \leftrightarrow \nu_s$  solution (see *e.g.* Ref. <sup>26</sup>) has two main features as compared with  $\nu_\mu \leftrightarrow \nu_\tau$ .

1). The neutral current interactions are suppressed in the  $\nu_\mu \leftrightarrow \nu_s$  case which can be realized by studying the reaction <sup>27,28</sup>

$$\nu N \rightarrow \nu N \pi^0. \quad (4)$$

At the SK, “ $\pi^0$ ” - events can also be generated by the charged current reactions, *e.g.*,  $\nu_\mu N \rightarrow \mu N' \pi^0$ , when muon is below the Cherenkov threshold (similar is for electron neutrinos). The  $\nu_\mu - \nu_\tau$  oscillations only weakly suppress the number of “ $\pi^0$ ”, whereas  $\nu_\mu - \nu_s$  can suppress the “ $\pi^0$ ” rate by 20 - 30 %. Preliminary SK data for 736 days give 279 “ $\pi^0$ ” events. This number even exceeds the expected number of events. To avoid normalization uncertainties one can consider the ratio of numbers of the “ $\pi^0$ ” - events and the  $e$ -like events:  $\pi^0/e$ . The experiment gives <sup>1</sup>

$$\frac{(\pi^0/e)_{data}}{(\pi^0/e)_{MC}} = 1.11 \pm 0.06(stat) \pm 0.02(MC \ stat) \pm 0.26(syst) \quad (5)$$

which is consistent with both channels of oscillations. Large systematic error in (5) is related to uncertainties in the cross-sections. Notice that the multi-pion production reactions give significant contribution to the  $\pi^0$  events (due to Cherenkov radiation threshold one or even more pions are not detected). In <sup>27</sup> the total uncertainty was estimated as being at the level 30%. The uncertainty will be diminished by direct measurements of the cross-section in the “forward” detector of the long baseline experiment K2K <sup>28</sup>. Another uncertainty is related to background, *e.g.*, from interactions of neutrons in the detector. For the  $\pi^0$  events this background is much more significant than for the  $e$ -like events since only in 17% of cases  $\pi^0$  will induce the  $e$ -like event.

It is also possible to study the zenith angle dependence of the  $\pi^0$ -events which is free of the uncertainties in the cross-section <sup>29</sup>. Another suggestion is to study the up-down asymmetry of the inclusive multi-ring events <sup>30</sup>.

Of course, a detection of the tau leptons produced by the  $\nu_\tau$  would be direct way to identify the solution. However, the number of the expected events is rather small <sup>31</sup>, and it is difficult to identify them. The detection of  $\nu_\tau$  is the main issue of future long base-line experiments.

2). The zenith angle distributions of the high energy events are different for  $\nu_\mu \leftrightarrow \nu_\tau$  and  $\nu_\mu \leftrightarrow \nu_s$  due to matter effect in  $\nu_\mu \leftrightarrow \nu_s$  channel <sup>32,33,34</sup>. The  $\nu_\mu \leftrightarrow \nu_s$  gives flatter  $\Theta$ -distribution. Matter suppresses oscillations of high energy neutrinos. For the neutrinos whose trajectories cross the mantle of the Earth one expects the strongest oscillation effect at  $|\cos \Theta| = 0.4 - 0.5$  and rather weak oscillation effect for the upward-going muons with  $|\cos \Theta| > 0.7$ . The enhancement of oscillations occurs for the

core crossing trajectories in contrast with naive expectation of further suppression of oscillations due to larger density of the core. The enhancement is related to the parametric effect based on the fact that the phase of oscillations acquired in the mantle and in the core is about  $\pi$ . As the result the ratio (DATA/MC) has two shallow minima at  $|\cos \Theta| = 0.4 - 0.6$  and at  $|\cos \Theta| = 0.85 - 0.95$  with minimal oscillation effect at  $\cos \Theta \approx 0.8$ . In contrast, the  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations lead to monotonous increase of the suppression with  $|\cos \Theta|$ . For the same values of oscillation parameters the  $\nu_\mu \leftrightarrow \nu_\tau$  oscillation effect at  $\cos \Theta = -1$  can be two times larger than in the case of  $\nu_\mu \leftrightarrow \nu_s$  oscillations, if both are normalized to the same value at  $\cos \Theta = 0$ . The SK data prefer  $\nu_\mu \leftrightarrow \nu_\tau$  mode, and  $\nu_\mu \leftrightarrow \nu_s$  mode is disfavored at  $2\sigma$  level<sup>13</sup>.

Main signatures of the SMA solution of the solar neutrino problem are:

1). The distortion of the recoil electron spectrum which can be characterized by a unique parameter: the slope of the ratio  $R(E)$ :  $s \equiv d(\ln R)/dE$ . No sharp enhancement of the signal is expected at the high energy part of the spectrum unless large flux of the *hep* - neutrinos is introduced. In the lowest energy bins one expects slight turn down of the spectrum.

2). Strong regeneration effect<sup>35</sup> is expected for neutrinos whose trajectories cross the core<sup>36</sup> due to the parametric enhancement of oscillations<sup>37,38,39</sup> as it was realized first in<sup>40</sup> see also<sup>41</sup>. As the result for  $\Delta m^2 > 10^{-2} \text{ eV}^2$  the N/D asymmetry is mainly due to the signal in the vertical bin (fifth night bin N5:  $\cos \Theta = -0.8 \div -1.0$ )  $F_5$ . Using this feature one can find the correlation between the relative excess of the flux in the N5 bin and the night-day asymmetry:

$$\frac{F_5}{\bar{F}} - 1 \approx 5A_{N/D} , \quad (6)$$

where  $\bar{F}$  is the average flux during 24 hours and  $F_5$  is the flux in the fifth night bin. Thus, precise measurements of  $F_5/\bar{F}$  and  $A_{N/D}$  can give important test of the solution.

No significant enhancement of signal has been yet observed in the N5 bin: This can be reconciled with result on the N/D asymmetry at  $2\sigma$  level. Notice that in the best fit point of the total rates one predicts rather small values of the  $F_5/\bar{F} - 1 \sim (1 - 5)\%$  and  $A_{N/D} \approx (0.2 - 1.0)\%$ .

3) The Earth regeneration effect leads to seasonal variations of the flux<sup>42,43</sup> on the top of the seasonal variations due to the eccentricity of the Earth orbit (geometrical effect). In the first approximation the seasonal dependence can be understood taking into account that regeneration occurs mainly when neutrinos cross the core. In this case the seasonal effect is proportional to weight,  $\Omega$ , of the trajectories which cross the core. For the SK place one gets  $\Omega_S = 0$  in the summer, and the winter-summer asymmetry can be written as

$$A_{W/S} = \Omega_W \left( \frac{F_5}{\bar{F}} - 1 \right) = 5\Omega_W A_{N/D} . \quad (7)$$

Notice that in contrast with geometrical effect the regeneration is very small in the wide interval from March to October and the effect increases in the winter time<sup>43</sup>. This differs also from predictions of seasonal asymmetry in the case of the LMA solution (see below). No significant enhancement of variations with the threshold is expected.

### 3 Large

There are three different schemes with large mixing of three active neutrinos (fig. 3).

(i) Scheme with single large (maximal) mixing where only  $\nu_\mu$  and  $\nu_\tau$  are mixed strongly and mixings of the  $\nu_e$  flavor are small. In this case the solar neutrino problem can be explained by the SMA MSW conversion.

(ii) Bi-large (bi-maximal<sup>44</sup>) scheme when  $\nu_\mu$  and  $\nu_\tau$  are mixed strongly in the heavy mass eigenstate and  $\nu_e$  is strongly mixed with the orthogonal combination of  $\nu_\mu$  and  $\nu_\tau$  in two light eigenstates. In this case the solar neutrino problem can be solved by the LMA MSW solution or by Vacuum Oscillations. The extreme case when both mixings are maximal is called the bi-maximal scheme.

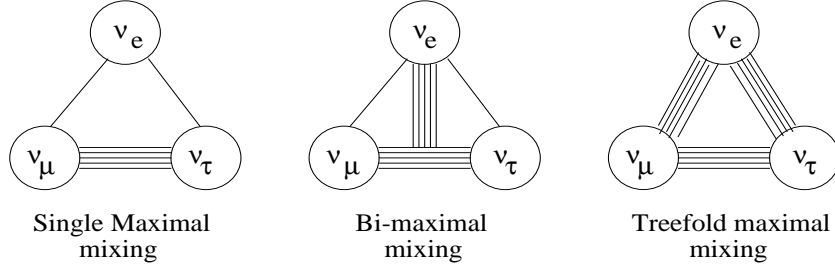


Figure 3: The large flavor mixing schemes.

(iii) Treefold maximal scheme<sup>45</sup>: All three neutrino flavors are mixed equally in the three mass eigenstates.

Let us consider phenomenological consequences of these schemes. Clearly, establishing the solution of the solar neutrino problem will play crucial role in identification of the mixing scheme.

### 3.1 Single Large (Maximal) Scheme

The spectrum is shown in fig. 4. Generic features of the scheme are  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations of the atmospheric neutrinos and the  $\nu_e \rightarrow \nu_\mu/\nu_\tau$  MSW conversion of the solar neutrinos. Signatures of these solutions have been already discussed in sect. 2. Namely, one predicts (i) no deficit of  $\pi^0$ -events (ii) sharp  $\Theta$ -dependence of the upward-going muons; (iii) distortion of the recoil electron spectrum produced by the boron neutrinos from the Sun with positive slope, etc..

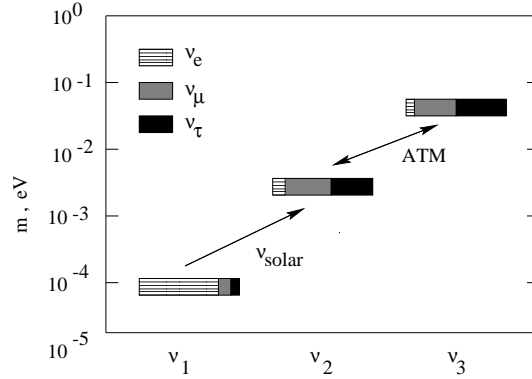


Figure 4: Neutrino mass spectrum with single large mixing.

To explain the LSND result one should introduce a sterile neutrino.

### 3.2 Bi-large mixing

The solar neutrino problem can be solved either by the Vacuum oscillations or by the LMA MSW conversion.

In the case of the VO solution (the spectrum is shown in fig. 5) the fit of the total event rates from existing experiments determines several disconnected regions of the oscillation parameters which can be classified by the phase of oscillations acquired by boron neutrinos with typical energy 10 MeV on the way from the Sun to the Earth. The best fit of the recoil electron spectrum can be obtained in the region with  $\phi = 5\pi/2$ :  $\Delta m^2 \approx 4 \cdot 10^{-10} \text{ eV}^2$  and  $\sin^2 2\theta = 0.8 - 1$ .

Typical shape of the recoil electron spectrum distortion  $R(E)$  is (i) the positive slope at  $E > 11$  MeV which can explain the excess of events (ii) shallow minimum at  $E \sim 8 - 10$  MeV and (iii) weak increase of the  $R(E)$  with decrease of the energy at  $E < 8$  MeV,  $R(E)$  approaches the constant value due to strong averaging effect. In the last series of observations a significance of the excess has further

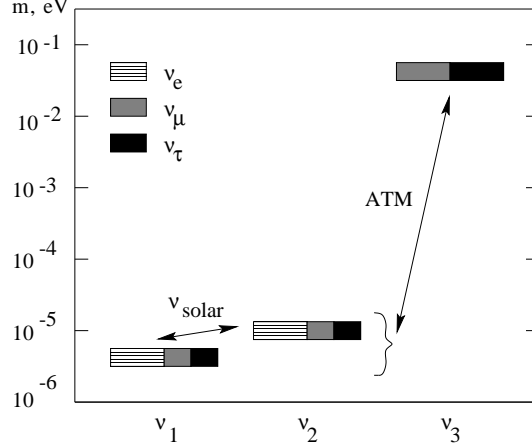


Figure 5: Neutrino masses and mixing pattern of the bi-maximal mixing scheme.

decreased which implies shift of the best fit point to smaller mixing angles:  $\sin^2 2\theta = 0.8 - 0.9$ . This, however, worsen the fit of total rates.

Reasonable agreement can be also achieved in the “ $\pi/2$  - region” with  $\Delta m^2 \approx 8 \cdot 10^{-11} \text{ eV}^2$ .

(ii) No Night-Day asymmetry and the zenith angle dependence of signal should be observed. The Earth regeneration effect is negligible.

(iii) The seasonal asymmetry is however expected on the top of pure geometrical effect ( $1/L^2$ ) due to eccentricity of the Earth orbit and the dependence of the oscillation probability on distance. It was noticed that seasonal variations<sup>46,47</sup> are correlated with spectrum distortion<sup>48 49</sup>. If the distortion in some part of the spectrum is characterized by the slope  $s \equiv d \ln R(E)/dE$ , the seasonal asymmetry of the events in the same part of the spectrum is proportional to:

$$A_{W/S} \propto s . \quad (8)$$

In particular, a positive slope corresponds to positive asymmetry which is in phase with pure geometrical effect ( $1/L^2$ ). So that the geometrical effect will be enhanced. Negative slope corresponds to negative asymmetry and compensation of the geometrical and oscillation effects. The  $5\pi/2$ -solution predicts almost flat distortion below 11 MeV and substantial distortion with a positive slope above 11 MeV. Therefore, the seasonal variations should be observed in the high energy part of the spectrum. This in turn, means that the relative asymmetry should increase with increase of the energy threshold<sup>12,51,50</sup>. As we marked, the data indicate such an enhancement, although the conclusion is statistically insignificant.

In the case of “ $\pi/2$  - region” one can expect a turn up of the spectrum at low energies (negative slope). Consequently, the negative asymmetry (due to oscillations) should be observed for the low energy events and the positive asymmetry – for the high energy events.

In “ $5\pi/2$ -domain” the oscillation length for the beryllium neutrinos ( $E \sim 0.86 \text{ MeV}$ ) is comparable with the change of the distance between the sun and the Earth during the year  $\Delta L$ , so that the change of the phase is

$$\Delta\phi = 2\pi \frac{\Delta L}{l_\nu} \sim 2\pi . \quad (9)$$

This means that one should observe about two periods of variations during the year and maxima can be in March-April and Sept- October<sup>51</sup>. These variations can be observed in experiments sensitive to the beryllium neutrino flux. In the Gallium experiments where main contribution comes from the  $pp$ -neutrinos the depth of variations of the Ge production rate due to variations of the Beryllium neutrino flux is about 30%<sup>51</sup>. Present statistics is not enough to establish these variations. Similar situation is in the Chlorine experiment. These variations can be established by the BOREXINO<sup>51</sup>.



Let us consider now the scenario with *bi-large mixing* (fig. 6) where the solar neutrino data are explained by the LMA MSW conversion of  $\nu_e$  to combination of  $\nu_\mu$  and  $\nu_\tau$ . The fit of the total rates and the Night-Day asymmetry gives the following range of the oscillation parameters:

$$\Delta m^2 = (1.4 - 18) \cdot 10^{-5} \text{eV}^2, \quad \sin^2 2\theta = 0.60 - 0.97. \quad (3\sigma) \quad (10)$$

Recent data contain some indications in favor of the LMA solution<sup>52</sup>:

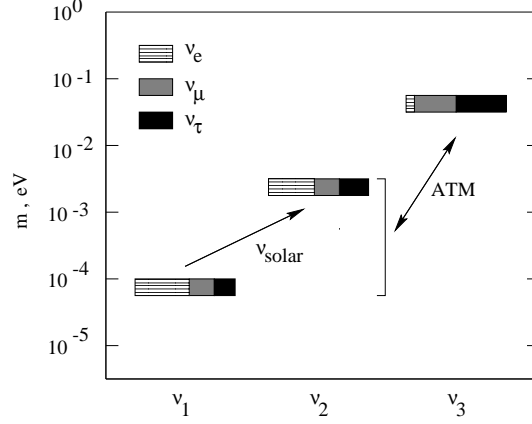


Figure 6: Neutrino masses and mixing pattern of the bi-large mixing scheme.

1). Solution predicts flat distortion of the spectrum with survival probability determined by vacuum mixing:  $P = \sin^2 \theta$ . For  $\Delta m^2 > 5 \cdot 10^{-5} \text{eV}^2$  the turn up of spectrum is expected at low energies due to effect of the adiabatic edge of the suppression pit. For  $\Delta m^2 < 3 \cdot 10^{-5} \text{eV}^2$  small positive slope appears due to the Earth regeneration effect. The excess of events in the high energy part of the spectrum, if confirmed, can be explained by the *hep* - neutrino contribution<sup>53</sup>.

The Earth matter regeneration leads to several related effects.

2). The Night-Day asymmetry. The asymmetry increases with decrease of  $\Delta m^2$ :

$$A_{N/D} \approx 0.22 \frac{10^{-5} \text{eV}^2}{\Delta m^2}, \quad (11)$$

and the dependence of  $A_{N/D}$  on the mixing angle is rather weak. Recent ( $1\sigma$ ) data can be explained if<sup>52</sup>

$$\Delta m^2 = (3.5^{+2.0}_{-1.5}) \cdot 10^{-5} \text{eV}^2. \quad (12)$$

3). The zenith angle distribution of events (averaged over the year) can lead to unique identification of the solution. The SK data show that the excess of events is not concentrated in the vertical night bin N5; the excess is observed already in the first night bin N1, so that the data indicate flat distribution. The LMA solution predicts flat distribution for  $\Delta m^2 > 2 \cdot 10^{-5} \text{eV}^2$  (fig. 7). Indeed, for these  $\Delta m^2$  the oscillation length in matter (being of the order of the vacuum oscillation length) is much smaller than the diameter of the Earth:

$$l_m < l^{res} \equiv \frac{l_\nu}{\sin 2\theta} = \frac{4\pi E}{\Delta m^2 \sin 2\theta} \ll d_{earth}. \quad (13)$$

Therefore integration over the zenith angle bin leads to averaging the oscillations. For smaller  $\Delta m^2$ :  $\Delta m^2 < 1.5 \cdot 10^{-5} \text{eV}^2$  and also smaller  $\theta$  the oscillation length becomes larger, averaging is not complete and the structure appears in the zenith angle distribution. This however happens in the range of parameters where the Night-Day asymmetry is large (practically excluded by experiment).

4) Seasonal asymmetry. Almost constant signal during the night allows one to establish simple relation between the Night-Day asymmetry (1) and the seasonal asymmetry (2). The asymmetry appears because the nights are longer in the winter than in the summer. Let  $F_N$  and  $F_D$  be the

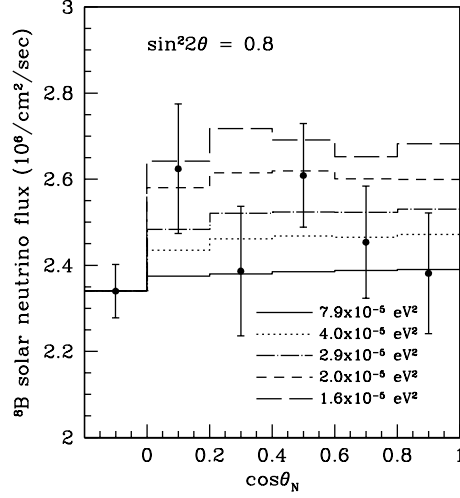


Figure 7: The zenith angle dependence of the total event rate above 6.5 MeV in the SuperKamiokande detector. Here  $\Theta_N = \pi - \Theta$  is the nadir angle. Also shown are the SK data from 708 days.

constant fluxes during the night and during the day. Consider the integrated flux (signal) during  $t_0 = 24$  hours. In winter this averaged signal equals

$$W = F_N t_W + F_D(t_0 - t_W) ; \quad (14)$$

in summer:

$$S = F_N t_S + F_D(t_0 - t_S) , \quad (15)$$

where  $t_W$  ( $t_S$ ) is the length of the night in Winter (Summer). Inserting these two expressions in to (2) and using the definition (1) we get after simple algebra:

$$A_{W/S} = A_{N/D} \frac{t_W - t_S}{t_0}. \quad (16)$$

Thus, the seasonal variations are proportional to the Night-Day asymmetry. For the SK location the time factor in (16) is about 1/6. Thus the seasonal asymmetry due to regeneration is expected to be 6 times weaker than the N/D asymmetry. For  $A_{N/D} = (3 - 9)\%$  which corresponds to  $1\sigma$  range of the observed N/D effect we find  $A_{W/S} = (0.5 - 1.5)\%$  <sup>52</sup>.

There are several features of the seasonal asymmetry which will allow one to distinguish LMA regeneration case from other effects <sup>52</sup>.

(i) No seasonal variation of the day rate is expected in contrast with Vacuum Oscillation case. The same is also correct for the night rate.

(ii) The regeneration asymmetry and the geometrical effect are in phase in the northern hemisphere and they are in opposite phase in the southern hemisphere which leads to cancellation. Thus, the detector in southern hemisphere should see weaker (up to factor 1/2) seasonal variations than the detector in the northern hemisphere.

Independent test of the LMA solution follows from studies of atmospheric neutrinos <sup>54</sup>. Indeed, the  $\Delta m^2$  responsible for the LMA MSW solution can give an observable effect: the  $\nu_\mu \leftrightarrow \nu_e$ -oscillations

driven by solar  $\Delta m^2$  lead to the excess of  $e$ -like atmospheric neutrino events. The excess can be defined as

$$\epsilon_e \equiv \frac{N_e}{N_e^0} - 1, \quad (17)$$

where  $N_e$  and  $N_e^0$  are numbers of events with and without oscillations. Notice that for  $\Delta m^2$  in the range of the LMA solution the matter suppresses the depth of oscillations (effective mixing). The suppression weakens with increase of  $\Delta m^2$ , and correspondingly, the excess increases. In the allowed region of parameters it can reach  $\sim 10\%$ . There is a complementarity of searches for the N/D asymmetry of the solar neutrino signal and the excess of the  $e$ -like events in atmospheric neutrinos. According to fig. 8 (from<sup>55</sup>) with increase of  $A_{N/D}$  the excess decreases and vice versa. For the central value of the present N/D asymmetry the excess is rather small:  $(2 - 5)\%$ . The excess increases with  $|\cos \Theta|$ , leading to a positive up - down asymmetry. The asymmetry is very weak in the low energy part of the sub-GeV sample ( $p < 0.4$  GeV) but it is clearly seen in the high energy part ( $p > 0.4$  GeV) of the sample<sup>54</sup>. The excess decreases with increase of energy of events: it is practically absent in the multi-GeV sample. New high precision and high statistics experiments are needed to establish this effect.

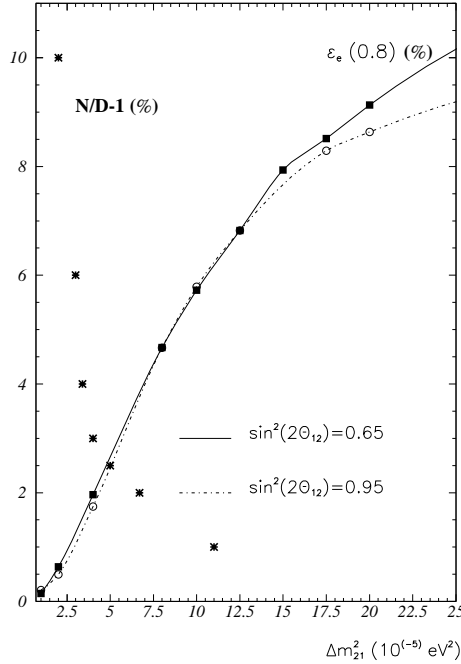


Figure 8: The excess of the  $e$ -like events as the function of solar  $\Delta m^2$  for two different values of  $\sin^2 2\theta_{12}$  relevant for solar neutrinos and  $\sin^2 2\theta_{23} = 0.8$ . Also shown by crosses is the dependence of the Night-Day asymmetry on  $\Delta m^2$ .

One more remark: the LMA solution implies the mass of the second neutrino in the range  $(0.4 - 1.0) \cdot 10^{-2}$  eV which is only one order of magnitude smaller than the mass  $m_3$  relevant for the atmospheric neutrino anomaly:

$$m_2/m_3 \sim 10^{-1}.$$

All other solutions of the solar neutrino problem require stronger mass hierarchy. A weak hierarchy indicates large mixing. Indeed, the mass matrix with above ratio of the eigenstates can naturally lead to mixing angle  $\theta_\nu$ :

$$\tan \theta_\nu \sim \sqrt{\frac{m_2}{m_3}}. \quad (18)$$

This results in  $\theta_\nu = (18 - 25)^\circ$ . Diagonalization of the charge lepton matrix can give  $\theta_{cl} \sim \sqrt{\frac{m_\mu}{m_\tau}} = (10 - 13)^\circ$ , so that the total lepton mixing can be  $\theta_l = \theta_\nu + \theta_{cl} \approx (28 - 40)^\circ$  - close to maximal mixing. That is, the required large mixing can be obtained without special arrangements.

### 3.3 Threefold Maximal Mixing

In such a scheme<sup>45</sup> moduli of all the elements of the mixing matrix are assumed to be equal:  $|U_{\alpha i}| = 1/\sqrt{3}$ . All three frequencies of oscillations contribute to all flavor channels equally. In vacuum  $\nu_\mu \leftrightarrow \nu_e$  and  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations proceed with equal depth:  $\sin^2 2\theta = 4/9$ , so that the  $\nu_\mu$  - disappearance is characterized by  $\sin^2 2\theta = 8/9$ . Then the CHOOZ bound implies that  $\Delta m^2 < 10^{-3} \text{ eV}^2$ .

The solar neutrino survival probability equals  $P = 4/9 P_2 + 1/9$ , where  $P_2$  is the two neutrino oscillation probability with maximal depth and smallest mass splitting  $\Delta m_{12}^2$ . It is assumed that  $\Delta m_{12}^2 < 10^{-11} \text{ eV}^2$ , so that 1 - 2 subsystem of neutrinos is "frozen" and  $P_2 = 1$ . As the result, the solar neutrino flux has energy independent suppression  $P = 5/9$ .

In medium with large enough density  $V \gg \Delta m_{atm}^2/2E$  the channels of oscillations with  $\nu_e$  are strongly suppressed, and the oscillation pattern is reduced to  $\nu_\mu \leftrightarrow \nu_\tau$  oscillations with maximal depth. (Matter does not influence this system and since  $\nu_\mu$  and  $\nu_\tau$  enter the scheme symmetrically their mixing becomes maximal after decoupling of  $\nu_e$ .) This picture is a good approximation for the multi-GeV events and small  $\Delta m_{atm}^2$ . However, significant deviations from this picture appears for sub-GeV sample, where  $\nu_e$  channels turn out to be suppressed weakly. As the consequence, one expects significant excess of the  $e$ -like events in the sub-GeV range. The analysis shows that the threefold mixing scheme can not be excluded by atmospheric neutrino data and CHOOZ result at the level stronger than  $2\sigma$ <sup>56</sup> (final CHOOZ result can however change this number). In this scheme a description of the solar neutrino data is rather poor, unless one excludes the Homestake result from analysis.

## 4 Maximal

In the strict bi-maximal scheme one has  $U_{e3} = 0$ <sup>44</sup>. For large enough  $\Delta m^2$  the average survival probability for solar neutrinos is about 1/2. It was marked recently<sup>57</sup> that the Earth regeneration effect is non-zero even in the case of maximal mixing which leads to the Night-Day effect and to dependence of the survival probability on energy, and consequently, to distortion of the recoil electron spectrum measured at the SK<sup>57</sup>. The point is that the state which arrives at the earth is pure  $\nu_2$  state. Although this state has equal admixtures of the  $\nu_e$  and  $\nu_\mu$  and do not oscillate in vacuum it will oscillate in matter of the Earth since the eigenstates in medium do not coincide with mass states leading to non-trivial regeneration effect. The effect will be also in general case of incoherent fluxes of  $\nu_1$  and  $\nu_2$ , provided that their fluxes are different. The regeneration effect is absent only in the case of equal incoherent fluxes of  $\nu_1$  and  $\nu_2$ .

Reasonable description of the data can be obtained if the Homestake result is not included in the analysis.

For  $\Delta m^2$  in the range of VO solution the data disfavor maximal mixing which would lead to too strong distortion of the recoil electron spectrum.

Clearly, the strict maximal mixing implies certain symmetry which is difficult to implement in view of situation with quark masses and mixing and charge lepton masses.

## 5 Summary

With high confidence level we can say that the atmospheric neutrinos oscillate and these oscillations are due to non-zero neutrino mass. High statistics studies of the atmospheric and solar neutrinos will allow us to make significant step in reconstruction of the neutrino mass and flavor spectrum. The key elements of our analysis as far as atmospheric neutrinos are concerned, are

- identification of channel of neutrino oscillation;
- clarification of the role of the electron neutrinos.

Of course, further checks of the oscillation interpretation of the data will be continued.

2. Situation with solar neutrinos is still rather uncertain. Main goal – the identification of the solution is not yet reached. Different datasets favor different solutions and we still should select among vacuum oscillations, large mixing angle MSW, small mixing angle MSW solution etc..

The most important observations which can lead to breakthrough are

- Further measurements of the N/D asymmetry and zenith angle distribution of events. Here it will be important to include the information from the low energy bins 5.5 - 6.5 MeV and then 5.0 - 5.5 MeV.

- Measurements of spectrum distortion. Crucial results can be obtained from already operating SNO.

- KamLAND<sup>58</sup> experiment will allow to check LMA solution;

- BOREXINO<sup>59</sup> result can play decisive role in identification of solution.

3. Confirmation of the LSND anomaly would be the evidence of existence of sterile neutrinos (or light singlet fermion) which eventually can lead us far beyond the Standard Model.

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